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# FLOODING IN THE DEVELOPMENT SYSTEM WITH SQUARE HEAD-ON LOCATION OF VERTICAL INJECTION AND HORIZONTAL PRODUCTION WELLS

## INTRODUCTION

The influence of horizontal well systems and spacing upon productivity and upon current and ultimate oil recovery can be very significant. However, this question is hardly covered in the literature [1-3]. Analytical evaluations on the problem in question are rather important, because the optimization of horizontal well parameters and field development systems based on them for certain geological conditions of the reservoir allow the reduction of unjustified costs for development system construction without significant losses in their productivity.

## PROPOSED METHOD

Consider the problem in the following way: a homogeneous productive vertical-anisotropic reservoir with thickness h is working in the stationary mode. The development system is an areal square system with perfect vertical injection and horizontal production wells (with regard to reservoir penetration degree); the wellbore diameter is $2r_c$; head-on location of wells (fig. 1). The length of producing horizontal wells is $2\ell_0$, the distance between producers and injectors is L, $\chi = \sqrt{k_x / k_y}$ is vertical anisotropy of reservoir permeability, a – is a horizontal well direct axis offset with reference to the horizontal plane of reservoir symmetry; μ is the viscosity of flooding and displaced agents. Constant speed potentials $\varphi_i$ and $\varphi_p$ are maintained on the sides of injection and production wells.

Due to the symmetry of filtration area we restrict ourselves to the consideration of a fluid flow in the element of filtration area ABD showed in fig. 2.

Publication results [4-5] give us an opportunity to interpret the vertical well with radius $r_C$ as a flat cunette, $4r_C$ wide, with a high degree of accuracy. Under such a substitution, an error in system rate evaluation is zero, errors in flow speed field distribution are several orders smaller as compared with those for engineering calculations.

By means of sequential transformations of

$$z = \frac{L}{\sqrt{2}\, K\left(1/\sqrt{2}\right)}, \int_0^{\Im} \frac{d\Im}{\sqrt[4]{(1-\Im)^3\,(\Im+1)^3}}, \quad \Im = \frac{e}{2}\left(w + \frac{1}{w}\right), \tag{1}$$

where $e = \mathrm{sn}\left(\ell_0 K\left(1/\sqrt{2}\right)/L, 1/\sqrt{2}\right)\sqrt{1 + \mathrm{cn}^2\left(\ell_0 K\left(1/\sqrt{2}\right)/L, 1/\sqrt{2}\right)}$, having dropped modulus $1/\sqrt{2}$ with the purpose of simplification in complete elliptical integrals $K\left(1/\sqrt{2}\right)$ and elliptical functions $\mathrm{sn}\left(x, 1/\sqrt{2}\right)$ and $\mathrm{cn}\left(x, 1/\sqrt{2}\right)$ beforehand, we put plane W, where the flow in question is transformed into a linear-radial movement between circles of radiuses 1 and $r_M$, in correspondence with initial plane Z, where

$$r_M = \frac{2\,\mathrm{cn}\left(r_C K/L\right)}{e\,\mathrm{sn}^2\left(r_C K/L\right)} - \frac{\sqrt{e^2 \mathrm{sn}^4\left(r_C K/L\right) + 4\mathrm{cn}^2\left(r_C K/L\right)}}{e\,\mathrm{sn}^2\left(r_C K/L\right)}.$$

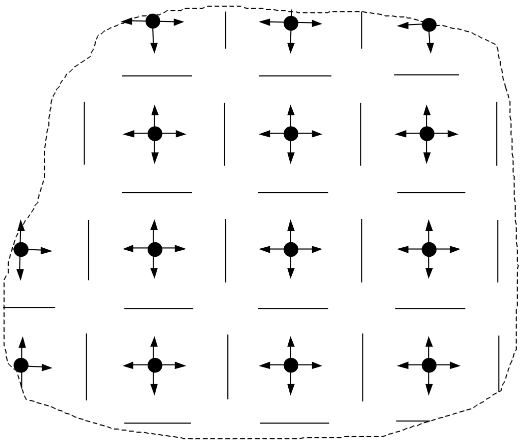

Figure 1 - Vertical and horizontal wells pattern

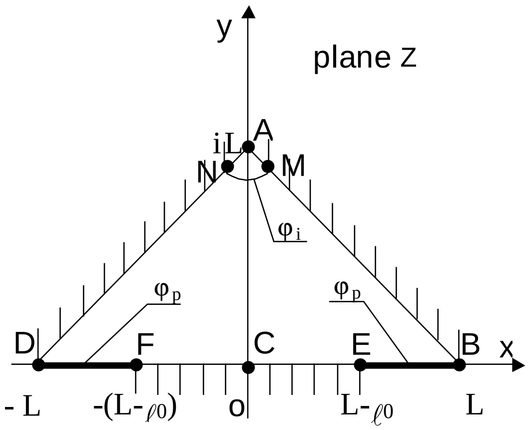


Figure 2 - Filtration element in plane Z

Then, the rate of the gallery in the development system in question is

$$Q = \pi\left(\varphi_Н - \varphi_Д\right) \Big/ \ln\left(1/r_M\right), \tag{2}$$

and rate (injectivity) of the well, which is imperfect with regard to gallery penetration degree, will be

$$Q_g = \frac{2\pi h\left(\varphi_Н - \varphi_Д\right)}{К_{imperf} \ln \dfrac{e\,\mathrm{sn}^2\left(r_C K/L\right)}{\sqrt{4\,\mathrm{cn}^2\left(r_C K/L\right) + e^2 \mathrm{sn}^4\left(r_C K/L\right)} - 2\,\mathrm{cn}\left(r_C K/L\right)}} \tag{3}$$

in accordance with [3] in the development system studied, where $К_{imperf} = 1 + \frac{h}{\tilde{L}}\frac{\chi}{\pi} \ln\left[\frac{h}{\pi r_C}\frac{\chi}{\chi+1}\cos^{-1}\frac{\pi a}{h}\right]$ – is the coefficient characterizing the imperfection degree of the horizontal well as compared with the gallery, $\tilde{L}$ is a weighted average length of current lines, where a potential loss takes place due to the trajectory distortion in the vertical direction close to the horizontal well, which value is defined by the following expression

$$\tilde{L} = \frac{1}{\ell_0} \int_{r_M}^{1}\int_{0}^{\pi} \sqrt{x_r^{/\,2}(r,\alpha) + y_r^{/\,2}(r,\alpha)}\, dr d\alpha, \text{ with } r = \sqrt{\mu^2 + \eta^2},\ \alpha = \arcsin(\eta / r).$$

The influence of well pattern parameters upon the relative productivity γ of the system in question, where γ is the relationship between filtration element rates with the area in the form of four symmetrically located horizontal wells, evaluated on (3), and the filtration system with a perfect external boundary of the reservoir with regard to reservoir penetration degree, in the form of a quadrate consisting of four galleries with side 2L, is presented in fig. 3.

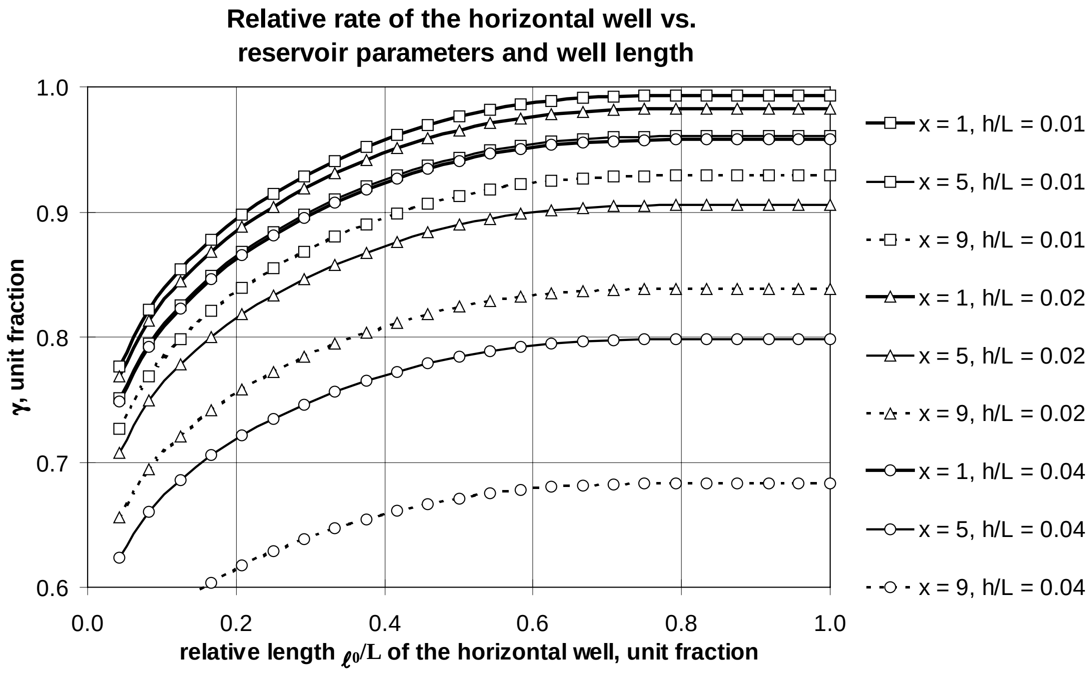


Figure 3 - Relative rate of the horizontal well vs. reservoir parameters and well length

The distribution of the potential in the half-circle of plane W is defined by the following expression

$$\varphi = \varphi_P + (\varphi_I - \varphi_P)\ln(r/r_M)/\ln(1/r_M). \quad (4)$$

As $V_Z = -d\varphi/dz$, then, differentiating (1) and (4), after intermediate transformations we will obtain an expression for the fluid flow speed

$$V_Z = \frac{\sqrt{2}\,K}{L\,K_{imperf}} \frac{\varphi_I - \varphi_P}{\ln(1/r_M)} \frac{cn^3(zK/L)}{\sqrt{1 - cn^4(zK/L) - e^2}}\ . \quad (5)$$

The distribution of fluid flow speeds at fluid filtration between the injector and horizontal production wells, when the length of the horizontal well is $\ell_0$ = 0.25L, is given in fig.4.

The integration of $\int_{z_1}^{z_2} 1/V_Z\,dz = \int_0^t dt$ of expression (5) allows the generation of the equation of phase division border progression at any moment of time t after the start of the injector operating, depending upon geometrical sizes of the filtration element.

$$\int_{r_M}^{r}\left[M(x,y)x_r^{/}(r,\alpha) - N(x,y)y_r^{/}(r,\alpha)\right]dr = t\,\frac{\sqrt{2}\,K\left(\varphi_{Н} - \varphi_{Д}\right)}{L\,K_{imperf}\ \ln\left(1/r_M\right)}, \qquad (6)$$

where M(x, y) = Re(1/$V_Z$), N(x, y) = Im(1/$V_Z$), $x_r^{/}(r,\alpha)$ и $y_r^{/}(r,\alpha)$ are derivatives of functions (1), connecting coordinates of the arbitrary point in planes Z and W, $r = \sqrt{\mu^2 + \eta^2}$, $\alpha = \arcsin(\eta / r)$.

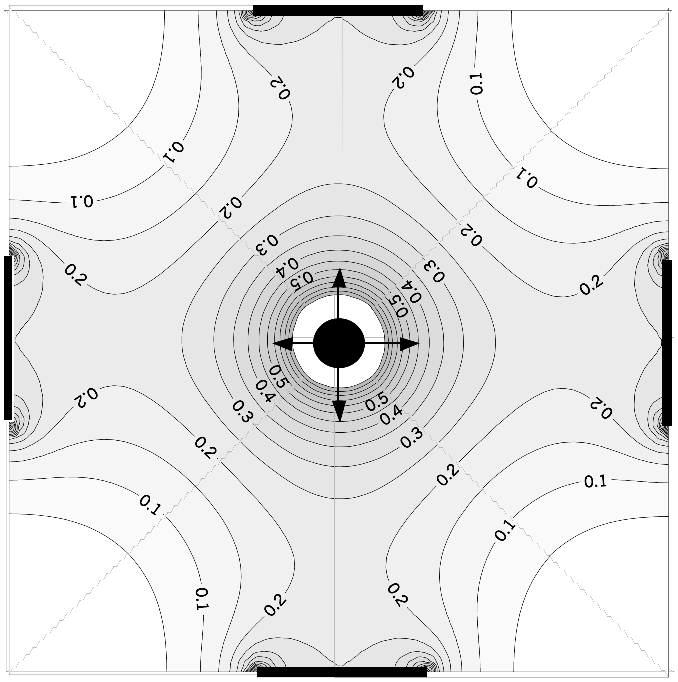


Figure 4 - Lines of equal fluid flow velocities in the development system in question

The breakthrough of the injection agent into a producing horizontal well takes place at moment $T_{wat}$ on the ordinate axis in the filtration element in Fig.2. Taking into account that r=1, after intermediate transformations, a joint solution of (1) и (6) gives an expression for the time of the water-free operation of the development system in question

$$T_{wat} = \frac{LK_{imperf}\ \ln\left(1/r_M\right)}{\sqrt{2}K\left(\varphi_I - \varphi_P\right)}\int_0^L cn\left(yK/L\right)\sqrt{1 - cn^4\left(yK/L\right)cn^4\left(\ell_0 K/L\right)}\,dy\,. \qquad (7)$$

Fig.5 shows the influence of the horizontal well length upon the relative time of the fluid breakthrough into the producing horizontal well. Time T is taken for the unit of the flat-radial fluid flowing between the external boundary of the reservoir with radius $R_k$=L, speed potential $\varphi_I$, and the vertical well with radius rc, speed potential $\varphi_p$; it is evidently expressed by the following relation:

$$T = L^2 \ln\left(L/r_C\right)/\left[2\left(\varphi_I - \varphi_P\right)\right].$$

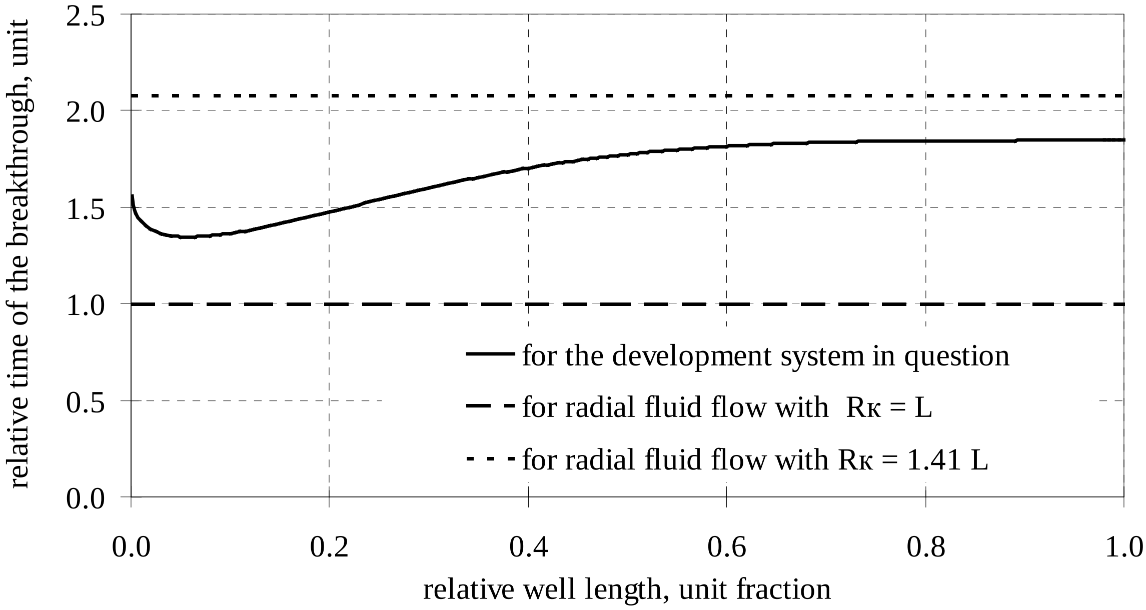


Figure 5 - Influence of geometric sizes of the filtration element and the well upon the relative time of fluid breakthrough into the HW

Analytical evaluations, presented in fig. 5 show that at the horizontal well length $\ell_0$=0.05-0.1L, the time of the displacing fluid breakthrough into the producer appears minimal. The existence of such a watering time extremum is explained by the following: even at a small increase in the horizontal well length as compared with the vertical well radius, the flowing character of the fluid along axis OY changes rather significantly – from radial to linear-parallel one, which is accompanied by a sharp reduction in filtration resistances and a corresponding growth of filtration rates. A further increase in horizontal well length leads to a significant re-distribution of velocities in the plane of fluid flow. The areas of high flow speeds are offset in different directions with reference to the fluid breakthrough line on axis OY. The flowing speed along the given axis goes down, the distant part of the reservoir is exposed to a harder drainage.

Let's evaluate the replacement factor at the moment $T_{wat}$ - the moment of the displacing agent breaking into the horizontal well. At substituting $K_z = QT_{wat} / 4L^2h$ for the displacement factor into the expression, a joint solution of (3) and (7) will produce the following formula for the evaluation of the displacement coefficient of the displaced liquid by a displacing fluid at moment $T_{wat}$ - the moment of the displacing agent forcing its way into the horizontal well.

$$K_z = \frac{\pi \ln\left(1/r_M\right)}{2\sqrt{2}\,LK} \frac{\int_0^L cn(yK/L)\sqrt{1 - cn^4(yK/L)cn^4(\ell_0 K/L)}dy}{\ln \frac{e\,sn^2(r_C K/L)}{\sqrt{4cn^2(r_C K/L) + e^2 sn^4(r_C K/L)} - 2cn(r_C K/L)}}. \quad (8)$$

The influence of the horizontal well length upon the replacement factor value of the displaced fluid, which is replaced by the fluid coming from the injection horizontal well, during the "water-free" period in shown in fig. 6. Value $\ell_0/L$ - the relationship of horizontal well length $\ell_0$ to the filtration element size L, is indicated on abscissa axis.

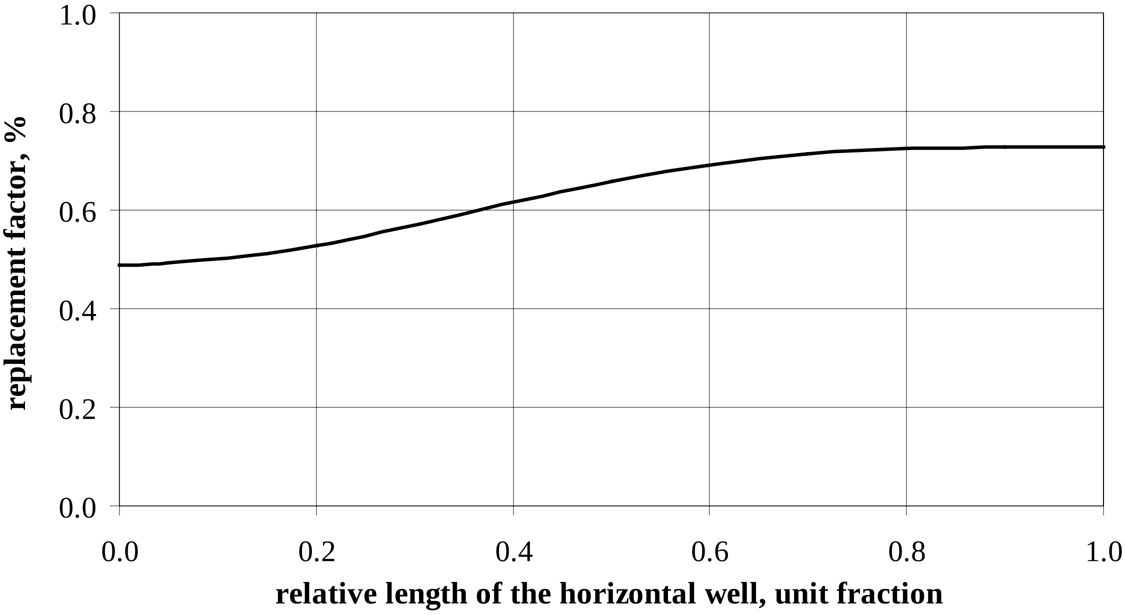


Figure 6 - Influence of horizontal well parameters and well pattern upon fluid replacement factor for non-water period

During the "water" period of horizontal well operation $t > T_{wat}$, some part of the horizontal wellbore will be working with a displacing agent, and the other one – with a displaced one. With the growth of system working time, an oil-water front will gradually move along the horizontal wellbore on axis OX from point C (0,0) to point E($\ell_0$,0). At the same time the proportion of the displacing agent in the well production

$$f_B(t) = \int_0^x V_z dx \Big/ \int_0^{\ell_0} V_z dx, \quad (9)$$

will grow respectively.

Due to the fact that the speed vector on a horizontal well side is rectangular to its axis, i.e. $V_z = V_{Y=0}$, taking into account (1), the joint solution of (6) and (9) will allow the generation of the expression for displacing agent $f_B$ in the production extracted

$$f_B(t) = \frac{2\sqrt{2}K\left(\frac{1}{\sqrt{2}}\right)}{\pi L} \int_0^{x(t)} \frac{cn^3(x(t)K/L)}{\sqrt{cn^4(x(t)K/L) - cn^4(\ell_0 K/L)}} dx \cdot \quad (10)$$

The dependence of the displacing agent content in the well production upon the volume of the fluid extracted from the reservoir for a number of lengths of horizontal wellbore sections is given in fig. 7.

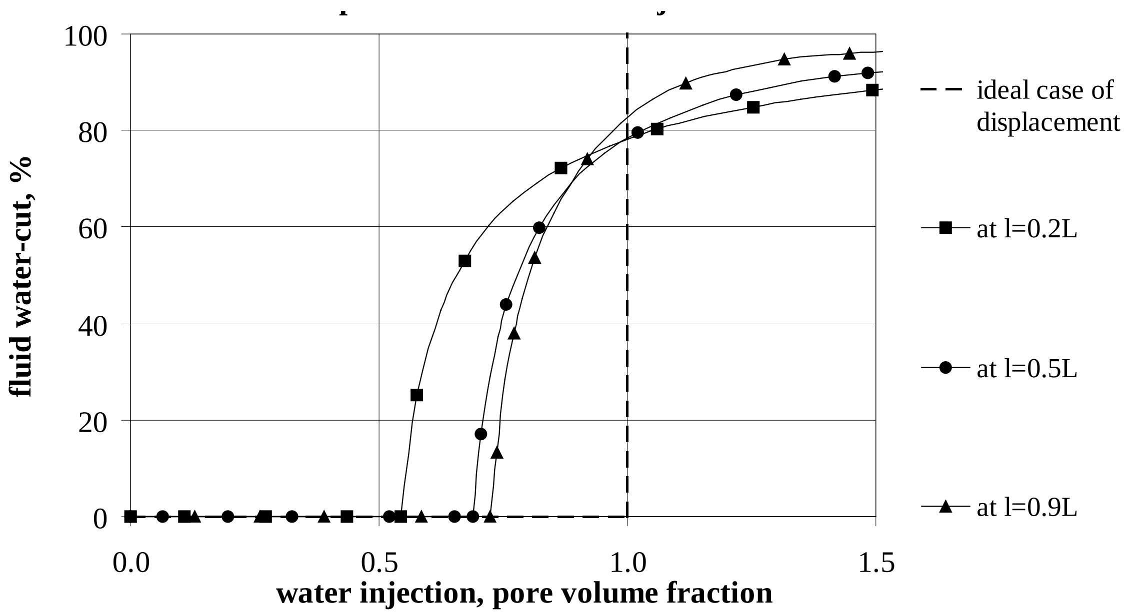


Figure 7 - Horizontal well production water-cut vs. pore volume of water injection

The “flushing” of reservoir $\sum \overline{Q}_{LIQ} = tQ/L^2h$, corresponding to time t (the volume of the injected fluid is expressed in pore volumes of the reservoir) is defined by the following expression

$$\sum \overline{Q}_{LIQ} = \frac{\pi t}{L^2 K_{imperf}} \frac{(\varphi_I - \varphi_P)}{\ln \dfrac{e\,sn^2(r_C K/L)}{\sqrt{4\,cn^2(r_C K/L) + e^2 sn^4(r_C K/L)} - 2\,cn(r_C K/L)}}. \qquad (11)$$

Dependence of the displaced agent cumulative production value expressed in reservoir pore volumes (reservoir “flooding” coefficient) upon the volume of the extracted fluid related to the reservoir volume (the so called dimensionless time of operation or reservoir flushing) in given in fig. 8 for a number of cases having different lengths of horizontal wells.

Similar to the behavior of production ”water-cut”, the replacement factor of the displaced agent tends to 1 at a never-ending flushing of the reservoir.

The progression of the fluid division front for different sequential moments of time t, which is described by equation (6) for $\ell_0/L = 0.2$, is given in fig. 9.

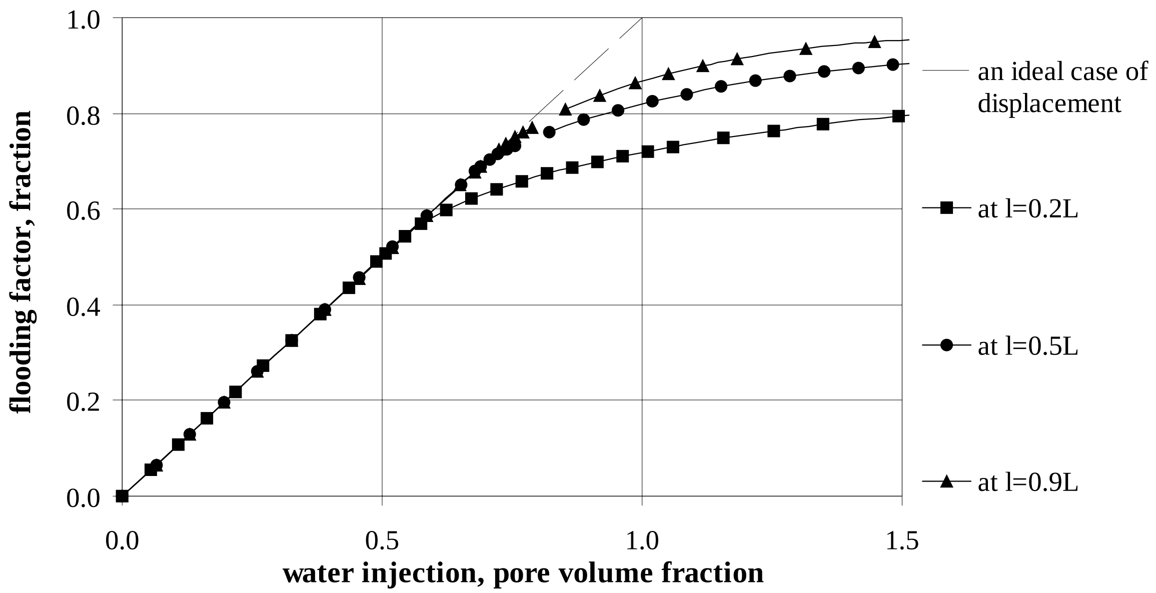


Figure 8 - Element flooding factor vs. pore volume of water injection

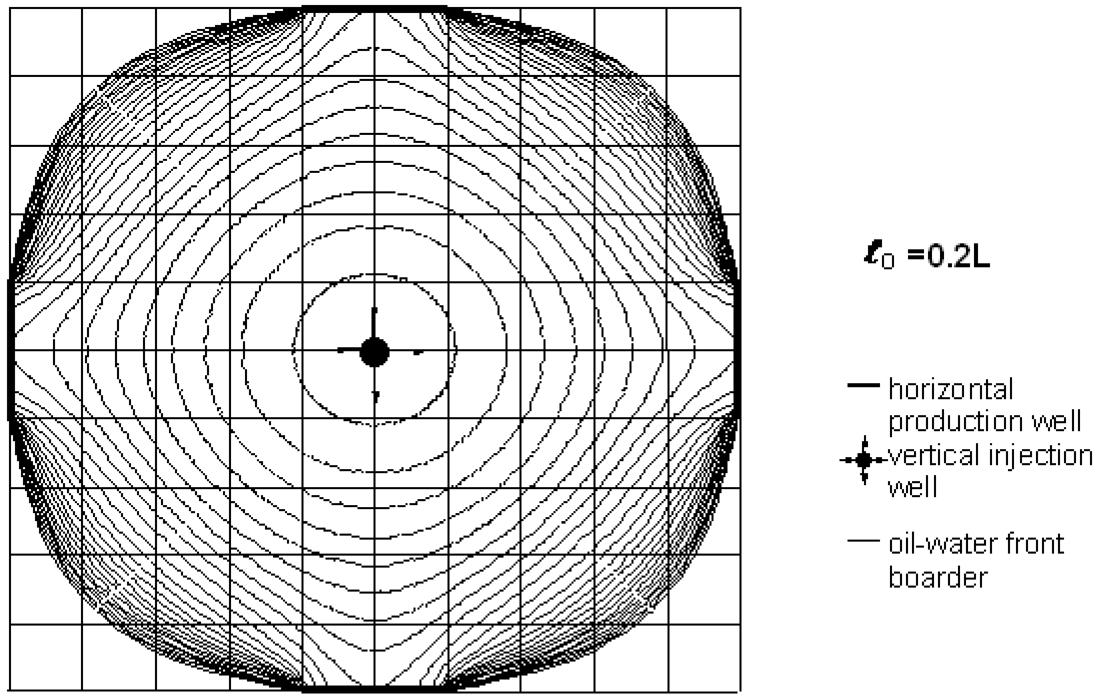


Figure 9 - Location of the division boarder between displacing agent and displaced one for sequential time points

**CONCLUSIONS**

- With the growth of the reservoir vertical anisotropy of permeability, the productivity of the development system in question can multiply decrease. At the same time when the geometrical size characteristic of the reservoir is $\chi h / L$<0.05, which is observed in most cases, the difference between horizontal well rate and perfect gallery rate will not exceed 5%.
- The growth of horizontal well length by more that 30-40 % of the characteristic size L of the well grid ceases to significantly affect the productivity of the development system in question.
- Breakthrough time of the fluid injected weakly depends upon the horizontal wellbore length.
- The designing of the horizontal well length lying in the range of more that 0.5 L will not significantly influence the water-free oil recovery, nor the rates of the further water-cut.

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