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CALCULATION OF RELATIVE PERMEABILITY FROM DISPLACEMENT TEST DATA

In the porous reservoir-rocks for the description and simulation of the two-phase flows the phase/relative permeability data and functions of fluids are necessary. The function of relative permeability is given generally depending on the degree of saturation of the wetting phase, and this function is determined in the most cases on the basis of data taken from laboratory displacement tests performed on rock-samples.

Displacement tests can be carried out through a relatively short time, but the evaluation of the data received can be considered as a highly complex job. During displacement measurement the processing of data is made difficult by the unfavorable capillary end-effect appearing on the inlet and outlet of the sample unless the displacement has been made by a great speed, because then the value of the end effect shall have been minimized.

Numerous methods are known to evaluate the data of displacement on an appropriately big speed, these often apply an auxiliary graphic method, and in other cases some empirical auxiliary functions will be applied.

The analytic method elaborated and developed by the Authors offers a possibility to describe the permeability functions more accurately almost semi-analytically, and to calculate these from the data of displacement.

INTRODUCTION

In the case of the description and simulation of the two-phase flow of the porous reservoir-rocks the knowledge of data and functions regarding relative permeability of fluids is necessary. The data of the relative permeability have been given generally on the dependence of saturation of the wetting phase. The functions of relative permeability - saturation can be determined most frequently on the basis of data of laboratory displacement tests carried out on rock samples. Regarding the determination of the values of relative permeability principally two laboratory methods have been introduced in the practice of the oil industry: a) method using the flow of steady-state condition, where both fluids considered as incompressible will be injected (simultaneously) into the core and the method b) using the flow of changing condition, where one of the fluids will be displaced by the other one, but only one fluid will be injected into the core simultaneously.

The data processing of tests using the steady-state flow is relatively simple, but it is difficult and complex to carry out the test itself because after each alteration in saturation the continuous average saturation along the length of the core should be kept on a steady value over a long time (several hours long). The laboratory test of displacement of changing flow condition can be performed within a relatively short time but the evaluation of data is a very complex job. In the case of both methods of measurement the unfavorable capillary end-effect appearing in the inlet and outlet sections make the processing of the data more complicated, unless the displacement was made by a not big speed, through which the rate of the end-effect will be minimized. It is difficult to reach a high speed, in the case a practical and an in-field displacement it can not be realized either.

Though in the case of all methods being known so far the parameters of a discrete measurement point have been applied for the calculation of the relative permeability, the method proposed by us which will be introduced in the following, however uses some functions determined on the basis of some mating parameters of displacement process to describe functions of the relative permeability.

THEORETICAL EXAMINATION OF THE TWO-PHASE DISPLACEMENT PROCESS WITH CHANGING FLOW

The laboratory measurements of displacement are generally carried out on cylindrical rock-cores under a linear flow. During experimental measurement of linear displacement process: A) **p_1-p_2= Δp(t) = a constant depression**, or B) **q_i=q_i(t) = a constant injection rate** are generally used. In this correspondence also the theoretical examination will be performed for these both conditions separately. It will be demonstrated how the sum and ratio of the mobility of the fluids in the displacement are dependent on parameters of the displacement, and on the cumulative volume of the injected fluid (expressed generally as a quotient of the pore volume).

A) Description of Linear Two-Phase Flow At ΔP= ΔP(T) = Constant Depression

After the breakthrough of displacing phase the Darcy law will be written on the two-phase linear flow established along the full length of the rock core, this applies both for the displacing (d) and the displaced (k) phase, by neglecting the gradient of the capillary pressure (dP_c/dx)=0:

$$q_d = -kA\frac{k_{rd}}{\mu_d}\frac{dp}{dx} \quad , \tag{1}$$

$$q_k = -kA\frac{k_{rk}}{\mu_k}\frac{dp}{dx} \quad . \tag{2}$$

By adding the (1) and (2) equations the

$$q_d + q_k = q_i = -kA\left(\frac{k_{rd}}{\mu_d} + \frac{k_{rk}}{\mu_k}\right)\frac{dp}{dx} \quad , \tag{3}$$

will be received. The function

$$Y(S_d) = \frac{k_{rd}}{\mu_d} + \frac{k_{rk}}{\mu_k} \tag{4}$$

will be introduced that should be the function of saturation of displacing phase, so the connection

$$q_i = -k\ A\,Y(S_d)\frac{dp}{dx} \tag{5}$$

will be given. After reordering the equation (5) the next differential equation will be given:

$$-dp = \frac{q_i}{kA}\frac{1}{Y(S_d)}dx \quad . \tag{6}$$

Integrating the given differential equation at the following conditions:

at x=0 $\quad$ $p=p_1(t)$ = const.;

at x=L $\quad$ $p=p_2(t)$ = const. and

$p_1 > p_1$

the next will be given:

$$p_1 - p_2 = \Delta p = \frac{q_i}{kA}\int_0^L \frac{1}{Y(S_d)}dx \,. \tag{7}$$

The solution of equation (7) to **Y(S$_d$)** if **Δp=constant**, when in the outlet the saturation of the momentary fluid of displacement is **S$_{d2}$** and **V$_i$(t)= ∫q$_i$dt**:

$$Y(S_{d2}) = \frac{L}{\Delta p\ k\ A}\frac{\left[\frac{dV_i(t)}{dt}\right]^3}{\left[\frac{dV_i(t)}{dt}\right]^2 + V_i(t)\frac{d^2V_i(t)}{dt^2}} \,. \tag{8}$$

B) Description of Linear Two-Phase Flow at constant Injection rate q_i=$q_i(t)$

From the moment of breakthrough the displacing phase **(t ≥ t$_a$)** a two-phase flow develop along the length of the whole linear core, for which in the case of **(dP$_c$/dx)=0** on the basis of the equation (7) beside **q$_i$=q$_i$(t)=constant** condition can be written, taking into consideration that **V$_i$(t)=q$_i$t and Δp=Δp(t)**, that is the pressure difference will change in time:

$$\Delta p(t) - t\frac{d\Delta p(t)}{dt} = \frac{q_i L}{k\ A}\frac{1}{Y(S_{d2})} \,. \tag{9}$$

By rearranging the equation (9) the function Y(S_{d2}) can be obtained in this case too,

$$Y(S_{d2}) = \frac{q_i L}{k\ A}\frac{1}{\left[\Delta p(t) - t\frac{d\Delta p(t)}{dt}\right]} \,. \tag{10}$$

By means of the connections (8) and (10) the function **Y(S$_d$)** can be determined in the outlet cross-section for both the displacement processes of the Δp(t)= constant depression and the q_i(t)=constant injection rate, that can be considered as the sum of mobility for the displacing and displaced fluids. In order to determine the value of the function it is required to know the time trend V_i=V_i(t) of the cumulative injected fluid during displacement, to know the time trend of Δp= Δp(t) of time pressure appearing on the rock sample as well as the knowledge of the geometrical dimensions and the absolute permeability of the core.

C) The Fluidum Proportions of The Flow of Changing Condition in the Outlet

In the both cases of **Δp = constant** and **q$_I$ = constant** for **t ≥ t$_a$** the following a linear equation can be written:

$$\frac{V_i(t)}{V_k} = a + b\left(\frac{V_i(t)}{V_p}\right) \,, \tag{11}$$

where

$a = f_{kf} = 1 - f_{df}$,

and **f$_{kf}$, f$_{df}$** - is the fraction of the displaced and displacing fluids at the breakthrough,

$b = 1/(S_{d\,max} S_{di})$,

S$_{dmax}$ - maximum saturation of displacing fluid reached during an infinite time displacement,

S$_{di}$ - the displacing fluid saturation of the rock-core at the beginning of the displacement.

According to the equation (11) the fluid rates in the outlet section after the break-through are

$$f_d = \frac{q_d}{q_k} = 1 - f_k \quad ; \tag{12}$$

$$f_k = \frac{q_k}{q_i} = \frac{a}{\left(a + b\frac{V_i(t)}{V_p}\right)^2} \tag{13}$$

and thus also the rate of mobility of both fluids can be written by means of the equations (1), (2), (12) and (13) for the outlet section:

$$M_{d2} = \frac{q_d}{q_k} = \frac{f_d}{f_k} = \frac{1-f_k}{f_k} = \frac{1}{f_k} - 1 = \frac{\left[a + b\frac{V_i(t)}{V_p}\right]^2}{a} - 1 . \tag{14}$$

D) The Equations Of Calculation of the Relative Permeability

Previously by means of the connections conducted according to the Par. A, B, and C, knowing the data of the displacing process the sum of mobility of both the displacing and displaced fluids $\mathbf{Y(S_{d2})}$ can be determined in the outlet section and the ratio of the mobility of both the displacing and the displaced fluids $\mathbf{M_{d2}}$ as well as the values of the relative permeability, too. Having these data the mobilities and the relative permeabilities of the displacing and displaced phases can be determined too. In order to determine the mobilities the next two equations can be used:

$$\frac{k_{rd}}{\mu_d} = \frac{M_{d2}Y(S_{d2})}{M_{d2}+1}, \tag{15}$$

$$\frac{k_{rk}}{\mu_k} = \frac{Y(S_{d2})}{M_{d2}+1} \quad . \tag{16}$$

From the equation (15) and (16) knowing the **?**$_{\mathbf{d}}$ and $\boldsymbol{\mu_k}$ the relative permeabilities can be calculated:

$$k_{rd} = \mu_d \frac{M_{d2}Y(S_{d2})}{M_{d2}+1} = \mu_d \; f_d \; Y(S_{d2}), \tag{17}$$

$$k_{rk} = \mu_k \frac{Y(S_{d2})}{M_{d2}+1} = \mu_k \; f_k \; Y(S_{d2}). \tag{18}$$

E) The Saturation of outlet section

Using the linear equation (11) the saturation of the outlet section and the average saturation of the rock-core can be determined for any injected cumulative volume. The average saturation can be calculated by the next equation:

$$\bar{S}_d - S_{di} = \frac{\frac{V_i(t)}{V_p}}{a + b\frac{V_i(t)}{V_p}}, \tag{19}$$

while the saturation at the outlet can be determined by following equation

$$S_{d2} = S_{di} + b\left(\overline{S}_d - S_{di}\right)^2 = S_{di} + b\left[\frac{\dfrac{V_i(t)}{V_p}}{a + b\dfrac{V_i(t)}{V_p}}\right]^2 \quad (20)$$

F) The Approximation Functions of Pressure Difference and the Cumulative Injected Fluid Volume and That of the Injecting Time

On the basis of experiences of displacement tests the time changes of pressure difference and that of the cumulative injected fluid volume can be described with the next approximation functions. In the case of displacement **q_i=const**. and **$t \geq t_a$** the timely altering pressure difference Δp(t) can be approximated by following equation

$$\Delta p(t) = a_1\left(\frac{q_i t}{V_p}\right)^{b_1} = a_1\left[\frac{V_i(t)}{V_p}\right]^{b_1}, \quad (21)$$

while in the case of **Δp=const**. the cumulative injected fluid volume can be approached by next equation

$$V_i(t) = a_2\, t^{b_2} . \quad (22)$$

Using these approximation functions the equations (8) and (10), the functions **$Y(S_{d2})$** can be written in the following form:

in the case of Δp = const.

$$Y(S_{d2}) = \frac{L\, a_2\, b_2^2 \left(\dfrac{V_p}{a_2}\right)^{(1-1/b_2)}}{\Delta p\; k\; A(2\,b_2 - 1)}\left(\frac{V_i(t)}{V_p}\right)^{(1-1/b_2)} \quad (23)$$

and **in the case of q_i =const**.

$$Y(S_{d2}) = \frac{q_i\; L}{k\; A\; a_1\,(1-b_1)\left(\dfrac{V_i(t)}{V_p}\right)^{b_1}} . \quad (24)$$

The applicability of the method discussed previously has been introduced by using our own displacement tests and by using the data of displacement tests published by the literature.

DESCRIPTION OF THE EXPERIMENTS

Numerous displacement tests have been carried out by our laboratories in the previous years. During these tests the displacement procedures with changing flow condition (phase change) on different, mainly of sandstone rock samples, were examined. The tests have been carried out on a laboratory temperature, the oil used is a mineral oil diluted by kerosene without gas, while the water was brine having different salt content in the imbibition direction because the tested stone-cores were always water-wet.

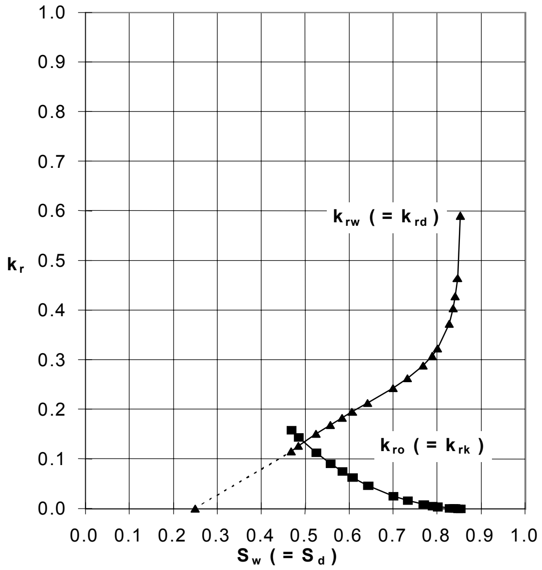

Fig. 1. Relative permeability curves (P rock sample, water injection, imbibition, Δp = const.)

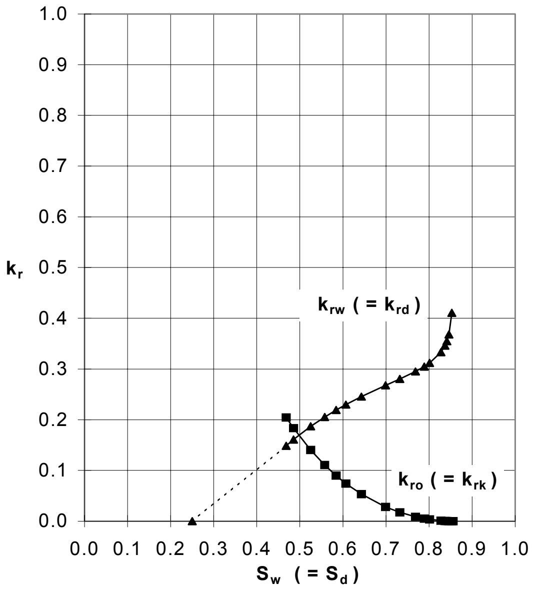

Fig. 2. Relative permeability curves (P rock sample, water injection, imbibition, q_i = const.)

Using the displacement data the parameter **a** and **b** of the linear displacement equation can be determined by a regression calculation. The parameters $\mathbf{a_1}$ and $\mathbf{b_1}$ in the case $\mathbf{q_I}$ **= const.**, and the parameters $\mathbf{a_2}$ and $\mathbf{b_2}$ for the case **Δp = const**. can also be calculated after the breakthrough of the displacing phase. By means of the functions $\mathbf{Y(S_d)}$, $\mathbf{f_d}$, and $\mathbf{f_k}$ the $\mathbf{k_r}$-values can be determined for a further non-dimensional $\mathbf{V_i/V_p}$ displacing phase.

The relative permeability function received on the basis of the processing of measured data of the **P**-type rock-core can be shown by the Figure 1. and 2.

The curves that can be seen on the Figure 3. are given on the basis of evaluation of data of the tests that had been carried out in the drainage direction on the rock-core **TJ**. On the basis of these curves it can be seen that in the case of carefully chosen displacing characters (phase change) the phase of displacement after the breakthrough can embrace a big range of saturation.

The phase change was also performed on drainage-direction on the rock core **D32/23**. The obtained curves of the relative permeability are shown by the Figure 4, the characteristic of displacement has also been performed appropriately, and by means of a measurement it was therefore possible to embrace a relatively wide range of saturation.

The results of rock-core **D32/30** have been shown by the $\mathbf{k_r}$-curves of the Figure 5.

The sections represented by dashed lines have been drawn through theoretical considerations and not calculations.

RESULTS AND DISCUSSION

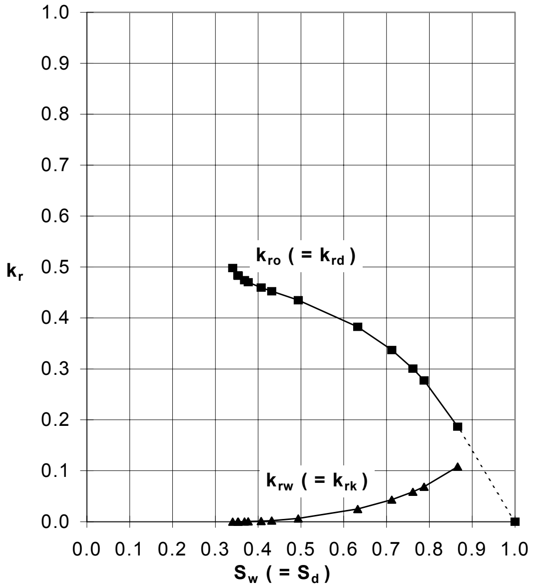

Fig. 3. Relative permeability curves (TJ-rock sample, oil injection, drainage Δp = const.)

During evaluation of laboratory displacement-tests of changing flow conditions on rock samples it should be always supposed that the rock-core is homogeneous and isotropic in flow direction because otherwise the tendencies of the results would not unequivocal or those could alter during the procedure of the exchange. If this happens then the evaluation of results can fail or the tendency change should be adapted, for example for the function of relative permeability received in result, which can cause problems of interpretation too. In the case of evaluation method described here it is practical to watch the next.

It results from theoretical and physical considerations that the next expressions should be used for the relative permeability functions controlling the exchange of the non-miscible fluid-phases:

in the case of drainage exchange the relative permeability of the wetting phase will reduce monotonously parallel with the reduction of the saturation of the wetting phase, and that of the non-wetting fluid will grow monotonously

in the case of imbibition exchange the relative permeability of the wetting phase will grow monotonously, and that of the non-wetting one however will reduce monotonously.

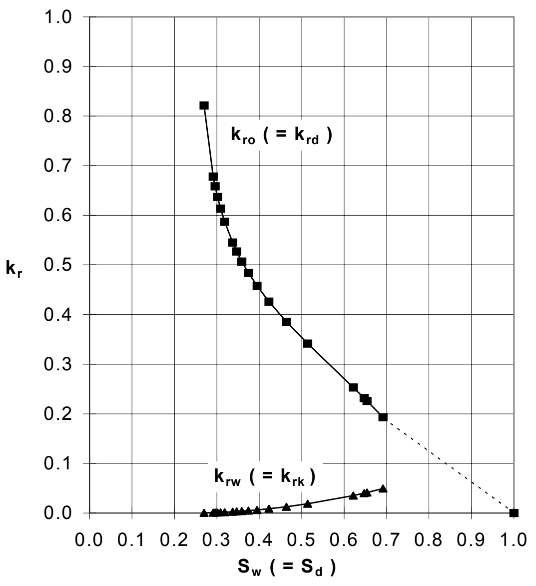

Fig. 4. Relative permeability curves (D32/23 rock sample, oil injection, drainage Δp = const.)

For the change of the phases (for the displacement process) either the constant yield $\mathbf{q_I = const}$. or beside a constant depression of $\mathbf{\Delta p = const}$. the quotient of the displacing and the displaced fluids (Leverett function) follows the next tendency:

in the drainage direction the fraction of the wetting phase ($\mathbf{f_k}$) will reduce monotonously from the moment of the breakthrough, that of the non-wetting phase will however grow monotonously,

during the imbibitions the fraction of the non-wetting phase will monotonously reduce from the breakthrough, but that of the wetting (displacing) phase ($\mathbf{f_d}$) will grow monotonously.

Concerning the exchange procedure of the phases beside both the $\mathbf{q_I = const}$. and $\mathbf{\Delta p=const}$. the linear function $V_i/V_k = a + b \cdot V_i/V_p$ is valid where the parameters $\mathbf{a=f_{kf}=1-f_{df}}$, $\mathbf{b=1/(S_{dmax}-S_{d0})}$ always have a positive value. Considering the above-mentioned it can be stated unequivocally that the procedure of a phase change beside

Δp = const. can only correspond with conditions discussed by us, if a relation $\mathbf{b_2 \geq 1}$ holds for the parameter $\mathbf{b_2}$, otherwise according to the equation (23) the function $\mathbf{Y(S_d)}$ does not reduce monotonously parallel with the rise of the function $\mathbf{(V_i/V_p)}$ but it will reduce.

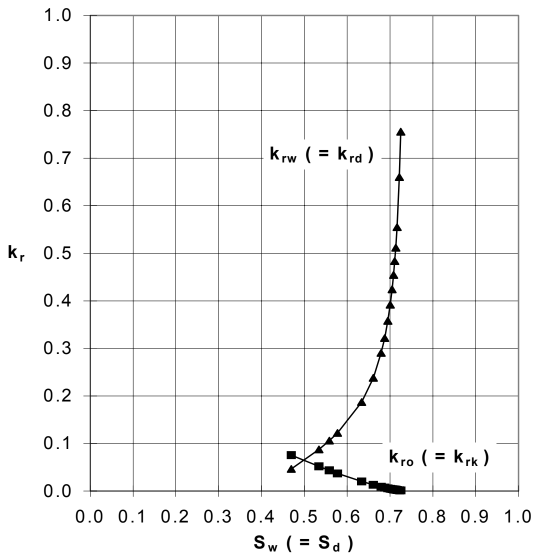

Fig. 5. Relative permeability curves (D32/30 rock sample, Water injection, imbibition, Δp = const.)

In the case of **-q_I = const**. the function **Y(Sd)** will only be monotonously growing equation (24), if a $\mathbf{b_1 \leq 0}$ relation holds for parameter $\mathbf{b_1}$. This tendency was demonstrated by P.M. Sigmund, and F.G. McCaffery too. It is unequivocally proven that the linear function described for the phase change: the $V_i/V_k = a + b \cdot V_i/V_p$, always describes the procedure from the moment of the break-through even in the case, if during the exchange also the capillary force gets role, however the equations having been given by the theoretical part - using the equations $\Delta p = a_1 (q_i\, t/V_p)^{b_1}$ or $V_i = a_2\, t^{b_2}$ will only describe a relative permeability for fluids included by the exchange, if the role of the capillary force is actually minimal in the procedure.

CONCLUSIONS

1. Regarding the change of altering flow of the non-miscible fluids, from the data of linear equation and those of the procedure following it in the case **Δp = const**. the connection $V_i = a_2\, t^{b_2}$ in the case of **q_I = const**. $\Delta p = a_1 (q_i\, t/V_p)^{b_1}$ obtained from the processing of the change of altering flow of non-miscible fluids, occurred as described, are appropriate to determine analytically the relative permeability curves relating the fluids.

2. The interrelations described by the theoretical part are simple enough to perform the calculation quickly and it is not necessary to use an auxiliary graphical procedure.

3. The function $Y(S_d) = (k_{rd}/\mu_d) + (k_{rk}/\mu_k)$ should be monotonously growing in both flow cases (including the necessity of having a constant value too), therefore the conditions $\mathbf{b_2 \geq 1}$ and $\mathbf{b_1 \leq 0}$ should be fulfilled in the equation (23) or (24) relating a given test. If these conditions would not hold, so the capillary force also had played a decisive role in the procedure of exchange.

SYMBOLS

$\mathbf{a, a_1, a_2}$	- constant values
A	- cross section of the rock-core, cm^2

b,b$_1$,b$_2$ - constant values
f - fraction (quotient), fraction
k - permeability, μm^2
k$_r$ - relative permeability, fraction
L - length of the rock-core, cm
M - rate of mobility, fraction
p - pressure, bar
Δp- pressure difference, bar
q - volume flow rate, cm^3/s
S - saturation, fraction
t - time, sec
V - cumulative volume, cm^3
Y(S$_d$) - auxiliary function, 1/mPa s
μ - dynamic viscosity, mPa s

Indices

a - breakthrough
d - displacing fluid
f - front
i - injected, initial
k - displaced fluid
p - pores
max - maximal
‾ - average
2 - outlet section